



ONE RAISED PRODUCT PRIME LABELING OF SOME PATH RELATED GRAPHS

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Abstract: One raised product prime labeling of a graph is the labeling of the vertices with $\{1, 2, \dots, p\}$ and the edges with product of the labels of the incident vertices plus 1. The greatest common incidence number of a vertex (*gcin*) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the *gcin* of each vertex of degree greater than one is one, then the graph admits one raised product prime labeling. Here we investigate some path related graphs for one raised product prime labeling.

Keywords: Graph labeling, product, prime labeling, prime graphs, path.

I. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol $V(G)$ and $E(G)$ denotes the vertex set and edge set of a graph G . The graph whose cardinality of the vertex set is called the order of G , denoted by p and the cardinality of the edge set is called the size of the graph G , denoted by q . A graph with p vertices and q edges is called a (p, q) - graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4]. Some basic concepts are taken from Frank Harary [1]. In this paper we investigated one raised product prime labeling of some path related graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (*gcin*) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. MAIN RESULTS

Definition 2.1 Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. Define a bijection $f : V(G) \rightarrow \{1, 2, \dots, p\}$ by $f(v_i) = i$, for every i from 1 to p and

define a 1-1 mapping $f_{orpp}^* : E(G) \rightarrow$ set of natural numbers N

by $f_{orpp}^*(uv) = f(u)f(v) + 1$. The induced function f_{orpp}^* is said to be one raised product prime labeling, if for each vertex of degree at least 2, the **gcin** of the labels of the incident edges is 1.

Definition 2.2 A graph which admits one raised product prime labeling is called one raised product prime graph.

Theorem 2.1 Path P_n ($n > 2$) admits one raised product prime labeling.

Proof: Let $G = P_n$ and let v_1, v_2, \dots, v_n are the vertices of G .

Here $|V(G)| = n$ and $|E(G)| = n-1$.

Define a function $f : V \rightarrow \{1, 2, \dots, n\}$ by

$$f(v_i) = i, i = 1, 2, \dots, n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$f_{orpp}^*(v_i v_{i+1}) = i^2 + i + 1 \quad i = 1, 2, \dots, n-1.$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \text{gcin of } (v_{i+1}) &= \text{gcd of } \{f_{orpp}^*(v_i v_{i+1}), f_{orpp}^*(v_{i+1} v_{i+2})\} \\ &= \text{gcd of } \{i^2 + i + 1, i^2 + 3i + 3\} \\ &= \text{gcd of } \{2i + 2, i^2 + i + 1\} \\ &= \text{gcd of } \{i + 1, i(i + 1) + 1\} \\ &= 1, \quad i = 1, 2, \dots, n-2. \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence P_n , admits one raised product prime labeling.

Example 2.1 $G = P_5$.

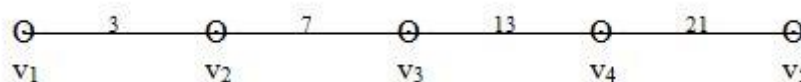


fig – 2.1

Theorem 2.2 Middle graph of Path P_n ($n > 2$) admits one raised product prime labeling.

Proof: Let $G = M\{P_n\}$ and let $v_1, v_2, \dots, v_{2n-1}$ are the vertices of G .

Here $|V(G)| = 2n-1$ and $|E(G)| = 3n-4$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n-1\}$ by

$$f(v_i) = i, i = 1, 2, \dots, 2n-1$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$f_{orpp}^*(v_i v_{i+1}) = i^2 + i + 1, \quad i = 1, 2, \dots, 2n-2.$$

$$f_{orpp}^*(v_{2i} v_{2i+2}) = (2i+1)^2, \quad i = 1, 2, \dots, n-2.$$

Clearly f_{orpp}^* is an injection.

$$\text{gcin of } (v_{i+1}) = \text{gcd of } \{f_{orpp}^*(v_i v_{i+1}), f_{orpp}^*(v_{i+1} v_{i+2})\}$$

$$= 1, \quad i = 1, 2, \dots, 2n-3.$$

So, *gcin* of each vertex of degree greater than one is 1.
Hence $M\{P_n\}$, admits one raised product prime labeling.

Example 2.2 $G = M(P_4)$

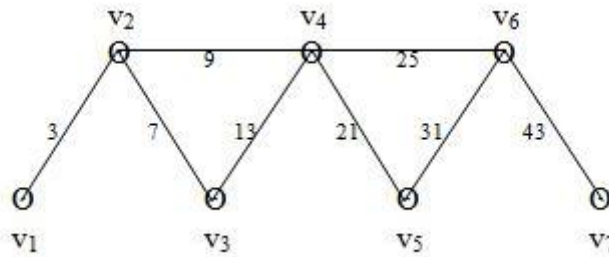


fig - 2.2

Theorem 2.3 Total graph of Path P_n ($n > 2$) admits one raised product prime labeling.

Proof: Let $G = T\{P_n\}$ and let $v_1, v_2, \dots, v_{2n-1}$ are the vertices of G .

Here $|V(G)| = 2n-1$ and $|E(G)| = 4n-5$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n-1\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n-1$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$f_{orpp}^*(v_i v_{i+1}) = i^2 + i + 1, \quad i = 1, 2, \dots, 2n-2.$$

$$f_{orpp}^*(v_{2i} v_{2i+2}) = (2i+1)^2, \quad i = 1, 2, \dots, n-2.$$

$$f_{orpp}^*(v_{2i-1} v_{2i+1}) = 4i^2, \quad i = 1, 2, \dots, n-1.$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{orpp}^*(v_1 v_2), f_{orpp}^*(v_1 v_3)\} \\ &= \text{gcd of } \{3, 4\} = 1. \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{i+1}) &= \text{gcd of } \{f_{orpp}^*(v_i v_{i+1}), f_{orpp}^*(v_{i+1} v_{i+2})\} \\ &= 1, \quad i = 1, 2, \dots, 2n-3. \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{2n-1}) &= \text{gcd of } \{f_{orpp}^*(v_{2n-3} v_{2n-1}), f_{orpp}^*(v_{2n-2} v_{2n-1})\} \\ &= \text{gcd of } \{(2n-2)^2, 4n^2 - 6n + 3\} \\ &= \text{gcd of } \{2n-2, (2n-2)(2n-1)+1\} = 1. \end{aligned}$$

So, *gcin* of each vertex of degree greater than one is 1.
Hence $T\{P_n\}$, admits one raised product prime labeling.

Example 2.3 $G = T(P_4)$

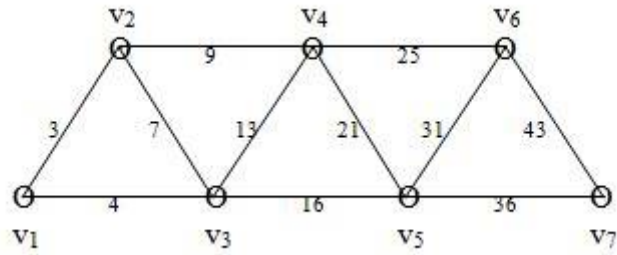


fig – 2.3

Theorem 2.4 Let G be the graph obtained by joining the corresponding internal vertices of two copies of path P_n .

G admits one raised product prime labeling.

Proof: Let G be the graph and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 3n-4$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n.$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$\begin{aligned} f_{orpp}^*(v_{2i-1} v_{2i+1}) &= (2i)^2, & i = 1, 2, \dots, n-1. \\ f_{orpp}^*(v_{2i} v_{2i+2}) &= (2i+1)^2, & i = 1, 2, \dots, n-1. \\ f_{orpp}^*(v_{2i+1} v_{2i+2}) &= 4i^2+6i+3, & i = 1, 2, \dots, n-2. \end{aligned}$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_{2i+1}) &= \gcd \{ f_{orpp}^*(v_{2i+1} v_{2i+3}), f_{orpp}^*(v_{2i+1} v_{2i+2}) \} \\ &= \gcd \{ (2i+2)^2, 4i^2+6i+3 \} \\ &= \gcd \{ 2i+2, (2i+1)(2i+2)+1 \} \\ &= 1, & i = 1, 2, \dots, n-2. \end{aligned}$$

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_{2i+2}) &= \gcd \{ f_{orpp}^*(v_{2i} v_{2i+2}), f_{orpp}^*(v_{2i+1} v_{2i+2}) \} \\ &= \gcd \{ (2i+1)^2, 4i^2+6i+3 \} \\ &= \gcd \{ 2i+1, (2i+1)(2i+2)+1 \} \\ &= 1, & i = 1, 2, \dots, n-2. \end{aligned}$$

So, \mathbf{gcin} of each vertex of degree greater than one is 1.

Hence G , admits one raised product prime labeling.

Example 2.4 Let G be the graph obtained by joining the corresponding internal vertices of two copies of path P_5 .

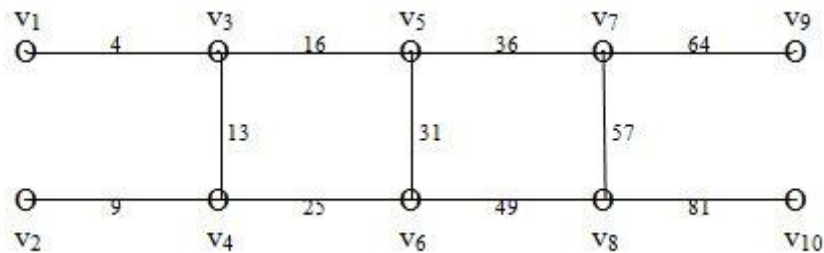


fig – 2.4

Theorem 2.5 Strong duplicate graph of path P_n admits one raised product prime labeling.

Proof: Let $G = S\{D(P_n)\}$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 3n-2$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$\begin{aligned} f_{orpp}^*(v_{2i-1} v_{2i}) &= 4i^2 - 2i + 1, & i = 1, 2, \dots, n. \\ f_{orpp}^*(v_{2i-1} v_{2i+2}) &= 4i^2 + 2i - 1, & i = 1, 2, \dots, n-1. \\ f_{orpp}^*(v_{2i} v_{2i+1}) &= 4i^2 + 2i + 1, & i = 1, 2, \dots, n-1. \end{aligned}$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_1) &= \gcd \text{ of } \{f_{orpp}^*(v_1 v_2), f_{orpp}^*(v_1 v_4)\} \\ &= \gcd \text{ of } \{3, 5\} = 1. \end{aligned}$$

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_{i+1}) &= \gcd \text{ of } \{f_{orpp}^*(v_i v_{i+1}), f_{orpp}^*(v_{i+1} v_{i+2})\} \\ &= 1, & i = 1, 2, \dots, 2n-2. \end{aligned}$$

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_{2n}) &= \gcd \text{ of } \{f_{orpp}^*(v_{2n-3} v_{2n}), f_{orpp}^*(v_{2n-1} v_{2n}), \} \\ &= \gcd \text{ of } \{4n^2 - 6n + 1, 4n^2 - 2n + 1\} \\ &= \gcd \text{ of } \{4n, 4n^2 - 6n + 1\} \\ &= \gcd \text{ of } \{n, 4n^2 - 6n + 1\} = 1. \end{aligned}$$

So, \mathbf{gcin} of each vertex of degree greater than one is 1.

Hence $S\{D(P_n)\}$, admits one raised product prime labeling.

Example 2.5 $G = S\{D(P_4)\}$

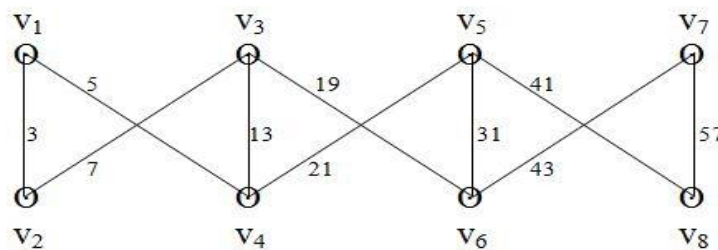


fig 2.5

Theorem 2.6 Z- graph of path P_n admits one raised product prime labeling.

Proof: Let $G = Z(P_n)$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 3n-3$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$f_{orpp}^*(v_{2i-1} v_{2i+1}) = 4i^2, \quad i = 1, 2, \dots, n-1.$$

$$f_{orpp}^*(v_{2i} v_{2i+1}) = 4i^2+2i+1 \quad i = 1,2,\dots,n-1.$$

$$f_{orpp}^*(v_{2i} v_{2i+2}) = 4i^2+4i+1 \quad i = 1,2,\dots,n-1.$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \text{gcin of } (v_{2i+1}) &= \text{gcd of } \{f_{orpp}^*(v_{2i-1} v_{2i+1}), f_{orpp}^*(v_{2i} v_{2i+1})\} \\ &= \text{gcd of } \{4i^2, 4i^2+2i+1\} \\ &= \text{gcd of } \{2i, 2i(2i+1)+1\} \\ &= 1, \quad i = 1,2,\dots,n-1. \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{2i}) &= \text{gcd of } \{f_{orpp}^*(v_{2i} v_{2i+1}), f_{orpp}^*(v_{2i} v_{2i+2}), \} \\ &= \text{gcd of } \{4i^2+2i+1, 4i^2+4i+1\} \\ &= \text{gcd of } \{2i, 2i(2i+2)+1\} \\ &= 1, \quad i = 1,2,\dots,n-1. \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence $Z(P_n)$, admits one raised product prime labeling.

Example 2.6 $G = Z(P_4)$

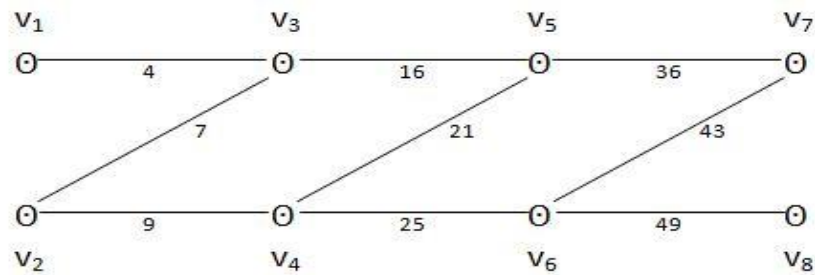


fig 2.6

Theorem 2.7 Shadow graph of path P_n admits one raised product prime labeling.

Proof: Let $G = D_2(P_n)$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 4n-4$.

Define a function $f : V \rightarrow \{1,2,\dots,2n\}$ by

$$f(v_i) = i, \quad i = 1,2,\dots,2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$f_{orpp}^*(v_{2i-1} v_{2i+1}) = 4i^2, \quad i = 1,2,\dots,n-1.$$

$$f_{orpp}^*(v_{2i-1} v_{2i+2}) = 4i^2+2i-1, \quad i = 1,2,\dots,n-1$$

$$f_{orpp}^*(v_{2i} v_{2i+1}) = 4i^2+2i+1, \quad i = 1,2,\dots,n-1.$$

$$f_{orpp}^*(v_{2i} v_{2i+2}) = 4i^2+4i+1, \quad i = 1,2,\dots,n-1.$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \text{gcin of } (v_1) &= \text{gcd of } \{f_{orpp}^*(v_1 v_3), f_{orpp}^*(v_1 v_4)\} \\ &= \text{gcd of } \{4, 5\} = 1. \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_2) &= \text{gcd of } \{f_{orpp}^*(v_2 v_3), f_{orpp}^*(v_2 v_4)\} \\ &= \text{gcd of } \{7, 9\} = 1. \end{aligned}$$

$$\begin{aligned}
 \text{gcin of } (v_{2i+1}) &= \text{gcd of } \{f_{orpp}^*(v_{2i-1} v_{2i+1}), f_{orpp}^*(v_{2i} v_{2i+1})\} \\
 &= \text{gcd of } \{4i^2, 4i^2+2i+1\} \\
 &= \text{gcd of } \{2i, 2i(2i+1)+1\} \\
 &= 1, \quad i = 1,2,\dots,n-1.
 \end{aligned}$$

$$\begin{aligned}
 \text{gcin of } (v_{2i+2}) &= \text{gcd of } \{f_{orpp}^*(v_{2i-1} v_{2i+2}), f_{orpp}^*(v_{2i} v_{2i+2}), \} \\
 &= \text{gcd of } \{4i^2+2i-1, 4i^2+4i+1\} \\
 &= \text{gcd of } \{2i+2, 4i^2+2i-1\} \\
 &= \text{gcd of } \{2i+1, 2i+2\} \\
 &= 1, \quad i = 1,2,\dots,n-1.
 \end{aligned}$$

So, **gcin** of each vertex of degree greater than one is 1.

Hence $D_2(P_n)$, admits one raised product prime labeling.

Example 2.7 $G = D_2(P_4)$

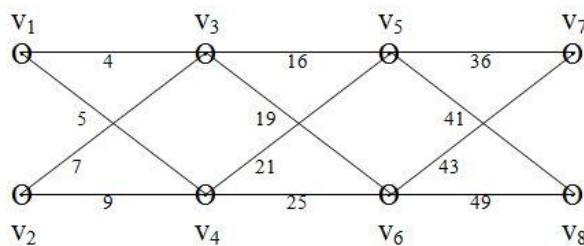


fig 2.7

Theorem 2.8 Strong shadow graph of path P_n admits one raised product prime labeling.

Proof: Let $G = S\{D_2(P_n)\}$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 5n-4$.

Define a function $f : V \rightarrow \{1,2,\dots,2n\}$ by

$$f(v_i) = i, \quad i = 1,2,\dots,2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$\begin{aligned}
 f_{orpp}^*(v_{2i-1} v_{2i}) &= 4i^2-2i+1, & i = 1,2,\dots,n-1. \\
 f_{orpp}^*(v_{2i-1} v_{2i+1}) &= 4i^2, & i = 1,2,\dots,n-1. \\
 f_{orpp}^*(v_{2i-1} v_{2i+2}) &= 4i^2+2i-1, & i = 1,2,\dots,n-1 \\
 f_{orpp}^*(v_{2i} v_{2i+1}) &= 4i^2+2i+1, & i = 1,2,\dots,n-1. \\
 f_{orpp}^*(v_{2i} v_{2i+2}) &= 4i^2+4i+1, & i = 1,2,\dots,n-1.
 \end{aligned}$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned}
 \text{gcin of } (v_1) &= \text{gcd of } \{f_{orpp}^*(v_1 v_2), f_{orpp}^*(v_1 v_3)\} \\
 &= \text{gcd of } \{3, 4\} = 1.
 \end{aligned}$$

$$\begin{aligned}
 \text{gcin of } (v_{i+1}) &= \text{gcd of } \{f_{orpp}^*(v_i v_{i+1}), f_{orpp}^*(v_{i+1} v_{i+2})\} \\
 &= 1, \quad i = 1,2,\dots,2n-2.
 \end{aligned}$$

$$\begin{aligned}
 \text{gcin of } (v_{2n}) &= \text{gcd of } \{f_{orpp}^*(v_{2n-2} v_{2n}), f_{orpp}^*(v_{2n-1} v_{2n})\} \\
 &= \text{gcd of } \{4n^2-4n+1, 4n^2-2n+1\} \\
 &= \text{gcd of } \{2n, 4n^2-2n+1\}
 \end{aligned}$$

$$= \text{gcd of } \{2n, 2n(2n-1)+1\}=1.$$

So, *gcin* of each vertex of degree greater than one is 1.

Hence $S\{D_2(P_n)\}$, admits one raised product prime labeling.

Example 2.8 $G = S\{D_2(P_4)\}$

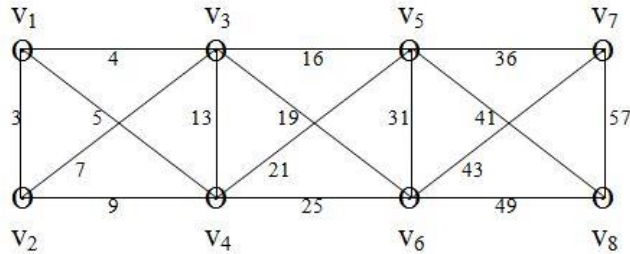


fig 2.8

Theorem 2.9 Splitting graph of path P_n admits one raised product prime labeling.

Proof: Let $G = S'(P_n)$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 3n-3$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$f_{orpp}^*(v_{2i-1} v_{2i+2}) = 4i^2+2i-1, \quad i = 1, 2, \dots, n-1$$

$$f_{orpp}^*(v_{2i} v_{2i+1}) = 4i^2+2i+1, \quad i = 1, 2, \dots, n-1$$

$$f_{orpp}^*(v_{2i} v_{2i+2}) = 4i^2+4i+1, \quad i = 1, 2, \dots, n-1$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \text{gcin of } (v_{2i+1}) &= \text{gcd of } \{f_{orpp}^*(v_{2i} v_{2i+1}), f_{orpp}^*(v_{2i+1} v_{2i+4})\} \\ &= \text{gcd of } \{4i^2+2i+1, 4i^2+10i+5\} \\ &= \text{gcd of } \{8i+4, 4i^2+2i+1\} \\ &= \text{gcd of } \{2i+1, 2i(2i+1)+1\} \\ &= 1, \quad i = 1, 2, \dots, n-2. \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{2i}) &= \text{gcd of } \{f_{orpp}^*(v_{2i} v_{2i+1}), f_{orpp}^*(v_{2i} v_{2i+2})\} \\ &= \text{gcd of } \{4i^2+2i+1, 4i^2+4i+1\} \\ &= \text{gcd of } \{2i, 2i(2i+1)+1\} \\ &= 1, \quad i = 1, 2, \dots, n-1. \end{aligned}$$

$$\begin{aligned} \text{gcin of } (v_{2n}) &= \text{gcd of } \{f_{orpp}^*(v_{2n-3} v_{2n}), f_{orpp}^*(v_{2n-2} v_{2n})\} \\ &= \text{gcd of } \{4n^2-6n+1, 4n^2-4n+1\} \\ &= \text{gcd of } \{2n, 4n^2-6n+1\} \\ &= \text{gcd of } \{2n, 2n(2n-3)+1\}=1. \end{aligned}$$

So, *gcin* of each vertex of degree greater than one is 1.

Hence $S'(P_n)$, admits one raised product prime labeling.

Example 2.9 $G = S'(P_4)$

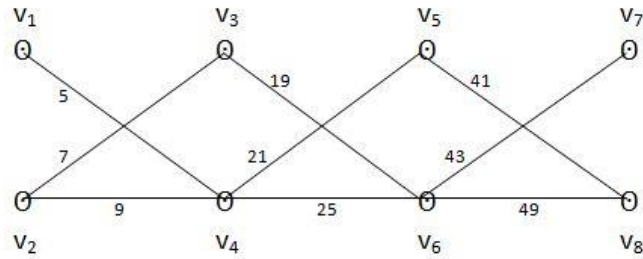


fig 2.9

Theorem 2.10 Strong splitting graph of path P_n admits one raised product prime labeling.

Proof: Let $G = S\{S'(P_n)\}$ and let v_1, v_2, \dots, v_{2n} are the vertices of G .

Here $|V(G)| = 2n$ and $|E(G)| = 4n-3$.

Define a function $f : V \rightarrow \{1, 2, \dots, 2n\}$ by

$$f(v_i) = i, \quad i = 1, 2, \dots, 2n$$

Clearly f is a bijection.

For the vertex labeling f , the induced edge labeling f_{orpp}^* is defined as follows

$$\begin{aligned} f_{orpp}^*(v_{2i-1} v_{2i}) &= 4i^2 - 2i + 1, & i = 1, 2, \dots, n. \\ f_{orpp}^*(v_{2i-1} v_{2i+2}) &= 4i^2 + 2i - 1, & i = 1, 2, \dots, n-1 \\ f_{orpp}^*(v_{2i} v_{2i+1}) &= 4i^2 + 2i + 1, & i = 1, 2, \dots, n-1 \\ f_{orpp}^*(v_{2i} v_{2i+2}) &= 4i^2 + 4i + 1, & i = 1, 2, \dots, n-1 \end{aligned}$$

Clearly f_{orpp}^* is an injection.

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_1) &= \gcd \text{ of } \{f_{orpp}^*(v_1 v_2), f_{orpp}^*(v_1 v_4)\} \\ &= \gcd \text{ of } \{3, 5\} = 1. \end{aligned}$$

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_{i+1}) &= \gcd \text{ of } \{f_{orpp}^*(v_i v_{i+1}), f_{orpp}^*(v_{i+1} v_{i+2})\} \\ &= 1, & i = 1, 2, \dots, 2n-2. \end{aligned}$$

$$\begin{aligned} \mathbf{gcin} \text{ of } (v_{2n}) &= \gcd \text{ of } \{f_{orpp}^*(v_{2n-3} v_{2n}), f_{orpp}^*(v_{2n-2} v_{2n})\} \\ &= 1. \end{aligned}$$

So, \mathbf{gcin} of each vertex of degree greater than one is 1.

Hence $S\{S'(P_n)\}$, admits one raised product prime labeling.

Example 2.10 $G = S\{S'(P_4)\}$

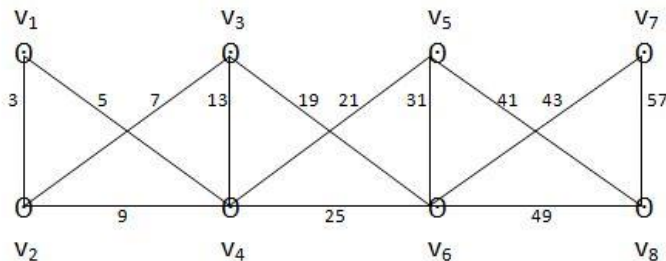


fig 2.10

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